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An LQR Algorithm to Orbit Maintenance of a Small Satellite Along a Circular Orbit

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Abstract—*In this paper, a linear quadratic regulator algorithm which minimizes the fuel consumption has been presented to orbit maintenance of a small satellite along a circular low earth orbit. The navigation and guidance system of this satellite consists of a GPS and six saturable thrusters in three plus and minus orthogonal directions. In order to estimate the orbit variations, an orbit propagator has been used that takes into account the effect of nonspherical earth and aerodynamic force. The results of this propagator have been verified by STK software. In this propagator, simulation of the atmosphere density, estimation of the atmospheric winds and calculation of the aerodynamic coefficients have been implemented using NRLMSIS-00, HWM-93, and analytical solution of the Boltzmann equation, respectively. Hence, after accurate prediction of orbital decay, the satellite orbit is controlled on both the orbital plane and its perpendicular plane by using this algorithm and sizing of the thrusters.*

1. INTRODUCTION

In recent investigations, navigation and orbit determination have usually been used interchangeably to mean determining the satellite's position and velocity or, equivalently, its orbital elements as a function of time. Similarly, both guidance and orbit control have been used to mean adjusting the orbit to meet some predetermined conditions. For satellites, orbit control has two important subsets. Orbit maintenance refers to maintaining the orbital elements but not the timing of when the satellite is at a particular location in the orbit. Stationkeeping refers to maintaining the satellite within a predefined box, which includes maintaining both the in-track position and the other orbital elements [1].

Altitude maintenance is an example of orbit maintenance in which occasional thruster firings are used to overcome drag and keep the orbit from spiraling downward. Geosynchronous stationkeeping maintains the satellite in a box over one place on the Earth. Stationkeeping in low-Earth orbit includes constellation maintenance, in which each satellite is maintained in a moving box defined relative to the rest of the satellites in the constellation [1].

Orbit maintenance is done by two methods: active and semi-active. In active method the required control force comes from the thrusters, whereas in semi-active method, the control force comes from the solar radiation pressure or drag force [2]. The thruster force is applied continuously or by impulse. In many cases it would be desirable to keep the thrusters active only for a small portion of each sampling period [3-5].

In this work, semi active method for orbit maintenance is done by continuous use of six orthogonal thrusters. For this purpose, the desired satellite is called the follower and a hypothetical satellite is used in an ideal orbit called leader. In this setup, having the position of leading satellite, the problem is changed to a tracking problem. Therefore, the responsibility of navigation and guidance system is to keep the follower orbit as close as possible to the leading satellite orbit. Solving this problem is a task which has some solutions, each with its specific characteristics [6], [7].

Since the equations of relative motion for leader and follower satellites in orbit plane and normal to it are decoupled, we can do the orbit maintenance for both separately. In this research, using a linear quadratic regulator, we can control orbit parameters in the orbit plane and in normal direction while minimizing fuel consumption. The studied orbit in this investigation is a circular LEO. (For more information about elliptical orbits see [8])

To design a guidance system to control a satellite orbit, we should predict precisely the orbit change and consequently choose suitable actuator and controller. Precise prediction of orbit change requires calculation of effective disturbance at the flight altitude. Since we assume small satellite at low earth altitude, the most important orbit disturbance is due to aerodynamics and nonspherical mass distribution of earth. Aerodynamics force calculation requires aerodynamic coefficients according to satellite geometry, density modeling at upper atmosphere, atmospheric winds simulation and future solar activity estimation [9].

In this paper, to calculate the aerodynamic coefficients, we divide the satellite body to flat planes and combine the results of Boltzmann equation solution. In upper atmosphere, density and atmospheric winds are functions of altitude, longitude, latitude, solar activity and time. Therefore, in this research, to estimate the density, we use NRLMSIS-00 and to model atmospheric winds for calculation relative velocity of satellite and atmosphere, we use HWM-93 [10-18].

Aerodynamic coefficient calculation requires attitude determination and control system (ADCS) modeling to determine the orientation of satellite with respect to upstream flow. The responsibility of the ADCS in this satellite is to keep the communication antennas toward earth using three orthogonal magnetorquers. This satellite has a cubic shape with a small magnetic boom on top surface, for solar panels on lateral surfaces and for communication antennas on the bottom surface. In this satellite, attitude determination is done using a 3D magnetometer and six solar sensors. Two of the solar sensors are on the top surface of the satellite and four of them are on the lateral surfaces. The reason for using two solar sensors on the top surface is that there is a possibility of being located in the shadow of the magnetometer boom. Also the reason for not using solar sensor on the bottom surface is to point that surface toward the earth [19-22].

2. DYNAMICS

Before designing the navigation and guidance system, it is necessary to know the satellite dynamics including both orbit dynamics and orientation dynamics. In orbit dynamics, the translation of the mass center is reviewed while in orientation dynamics, the attitude of satellite is studied. In this work to model satellite dynamics, we use inertial, earth centered coordinate system and Euler-Hale system [23]. The body and orbit coordinate systems are defined as in Fig. 1.

1. Orbit coordinate system (X_R, Y_R, Z_R) is an orthogonal coordinate centered at the mass center of the satellite. Z_R axis is in the direction of the line connecting the earth center to the mass center of the satellite. X_R axis is normal to the orbit plane and is in the direction of angular velocity of the orbit. Y_R axis is in the opposite direction of the satellite velocity and completes this right handed orthogonal coordinate system.

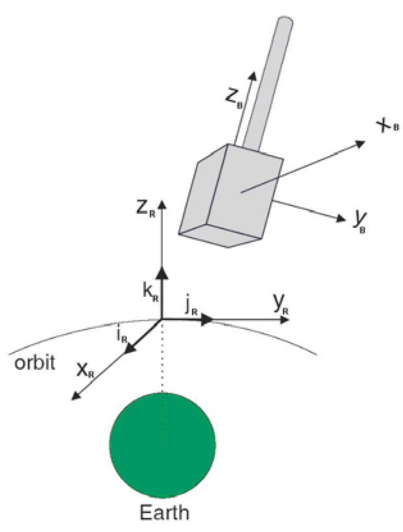

Fig. 1: The body and orbit coordinate systems.

2. The body coordinate system (X_B, Y_B, Z_B) is a right handed orthogonal coordinate system along the principle axes of the satellite centered in the mass center of the satellite. X_B axis is along the maximum moment of inertia axis and Z_B axis is along the minimum one. Y_B completes the right handed orthogonal coordinate system.

3. NAVIGATION SYSTEM

There are two types of orbit determination, differentiated by timing. Real-time orbit determination provides the best estimate of where a satellite is at the present time and may be important for spacecraft and payload operations, such as accurate pointing at some target. Definitive orbit determination is the best estimate of the satellite position and orbital elements at some earlier time. It is done after gathering and processing all relevant observations. Orbit propagation refers to integrating the equations of motion to determine where a satellite will be at some other time. Usually orbit propagation refers to looking ahead in time from when the data was taken and is used either for planning or operations. Occasionally orbits will be propagated backward in time, either to determine where a satellite was in the past or to look at historical astronomical observations in the case of comets or planets.

Traditionally, ground stations from around the world provide tracking data to a mission-operations center. When all data is available, definitive orbit determination provides the best estimate of the orbit. But nowadays, after operating GPS, GLONASS satellites or other autonomous orbit determination systems, instead of ground stations which are traditional systems, autonomous navigation systems are used that determine the satellite positions in real-time [1].

In this research the position of the satellite is determined using a GPS. To model its function, we use on orbit propagator which instead of integrating equations of motion uses orbital parameters. Satellite position and velocity change due to both earth gravity and disturbance forces. Orbital parameters change due to only disturbance forces therefore their rates of change are slower than those of position and velocity. Using this property, we can choose bigger integration steps and consequently minimizing calculation time [24].

However in circular and equatorial orbits, Keplerian orbital parameters have singular points. In this paper, in order to study the effect of these disturbances in these orbits, we make use of Equinoctial Parameters. Therefore, after position and velocity estimation, we add some noise due to the GPS used in the navigation system [25], [26].

4. GUIDANCE SYSTEM

In this investigation, to keep the follower orbit, we use a hypothetical satellite in a similar but ideal orbit which is called the leading satellite. Assuming circular orbit, the position vector of the follower with respect to Euler-Hale coordinate system attached to the leading satellite (x, y, z) is calculated based on the (1). In this equation, we assume small values for (x, y, z) compared to the orbit radius of the leading satellite.

$$\begin{aligned} &\ddot{x} - 2n\dot{y} - 3n^2 x = a_{xp} + a_{xc} \\ &\ddot{y} + 2n\dot{x} = a_{yp} + a_{yc} \\ &\ddot{z} + n^2 z = a_{zp} + a_{zc} \end{aligned} \tag{1}$$

In (1), n is angular velocity of the leading satellite, (a_{xp}, a_{yp}, a_{zp}) are the components of disturbance acceleration due to drag force and nonspherical mass distribution of earth and (a_{xc}, a_{yc}, a_{zc}) are the components of thruster control force divided by satellite mass which are applied on the follower satellite [23], [27].

$$X = [\dot{x}\ \dot{y}\ \dot{z}\ x\ y\ z]^T \tag{2}$$

$$\dot{X} = AX + Bu \tag{3}$$

$$A = \begin{bmatrix} 0 & 2n & 0 & 3n^2 & 0 & 0 \\ -2n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -n^2 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

Representation of (1) in state space assuming state vector as (2) would lead to the (3). Matrices A and B in (3) are computed from the (4) and (5).

Using linear quadratic regulator to minimize the performance measure mentioned in (5) to reduce the fuel consumption would lead to a gain matrix K which is explained in (6). In (5), R matrix is positive definite and symmetric and Q is the 6x6 identity matrix.

$$J = \int_0^\infty \left[X^T(t) Q X(t) + u^T(t) R u(t) \right] dt \tag{6}$$

$$u = -KX(t) \tag{7}$$

The K matrix is $R^{-1}.B^T.P$ and P is symmetric and positive definite obtained from algebraic Riccati equation (8) [28].

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{8}$$

5. SIMULATION RESULTS

The studied orbit in this investigation is LEO of 350 km altitude, circular and almost polar with an inclination of 83 deg. The dimensions of the satellite are 0.5m x 0.5m x 1m and its mass is 100 kg. The used thrusters saturate at ±5 N and the GPS has an error of ±10 m and ±1 m/s.

In Fig. 2 to 5, change in altitude, eccentricity, inclination, right ascension of the ascending node (RAAN) of the follower satellite without using any controller are shown. Fig. 6 to 9 show these parameters while using nonsaturating thrusters. These parameters have been shown in Fig. 10 to 13 while thrusters saturate at ±5 N. In Fig. 14 to 16 control acceleration of nonsaturating thrusters are shown.

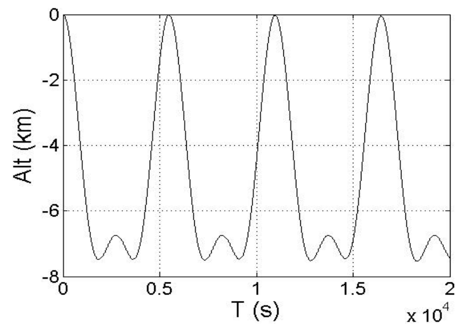

Fig. 2: Altitude variation without control

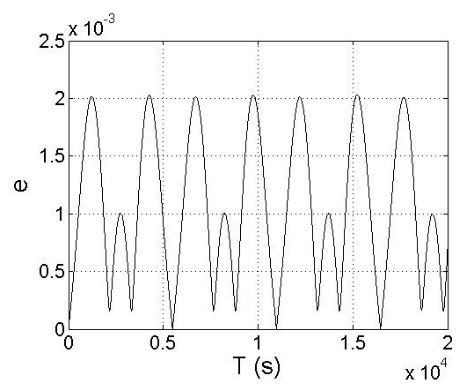

Fig. 3: Eccentricity variation without control

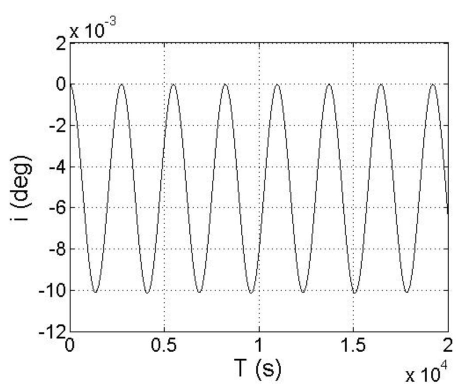

Fig. 4: Inclination variation without control

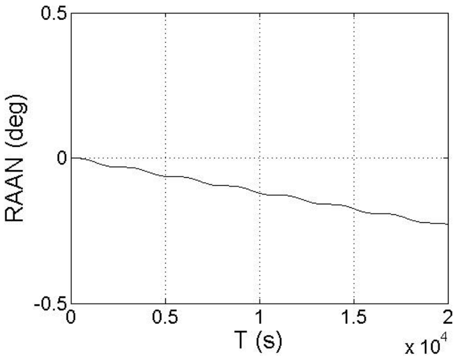

Fig. 5: RAAN variation without control

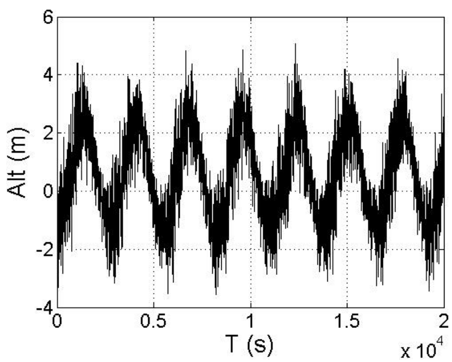

Fig. 6: Altitude Control without saturation

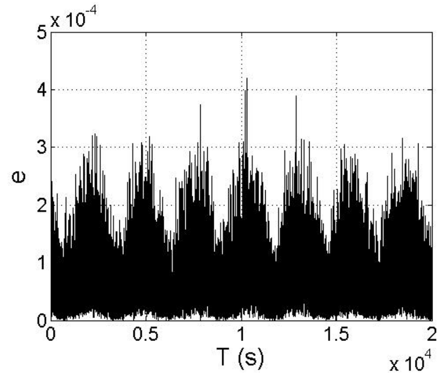

Fig. 7: Eccentricity Control without saturation

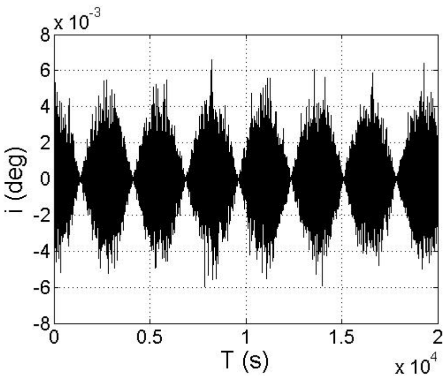

Fig. 8: Inclination Control without saturation

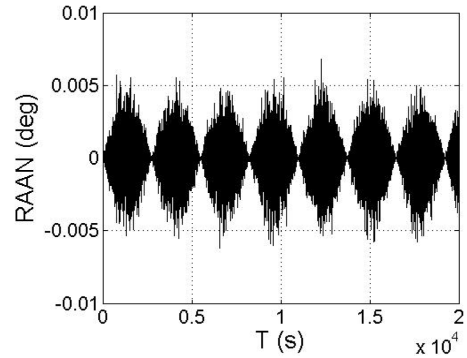

Fig. 9: RAAN Control without saturation

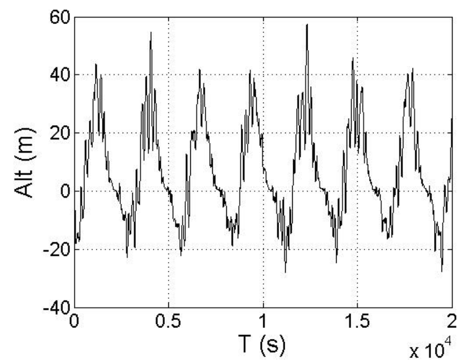

Fig. 10: Altitude Control with saturation

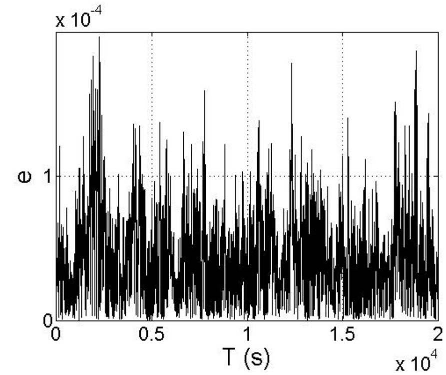

Fig. 11. Eccentricity Control with saturation

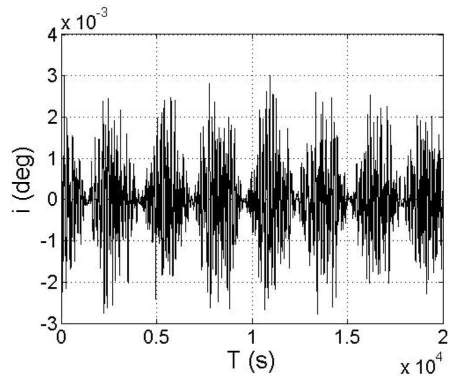

Fig. 12: Inclination Control with saturation

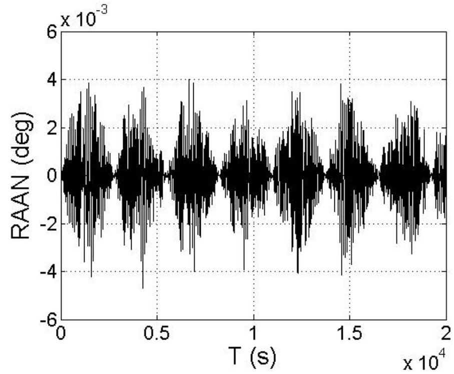

Fig. 13: RAAN Control with saturation

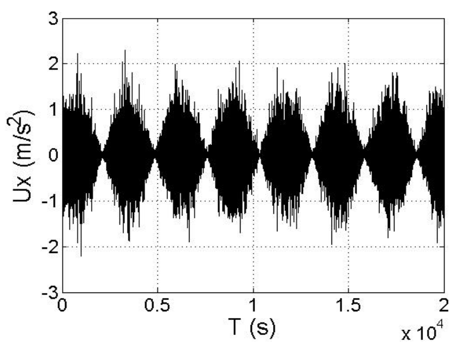

Fig. 14: X component of the control acceleration without saturation

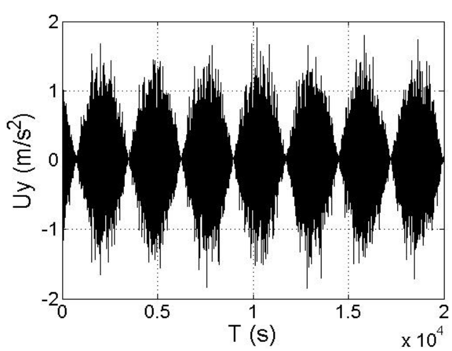

Fig. 15: Y component of the control acceleration without saturation

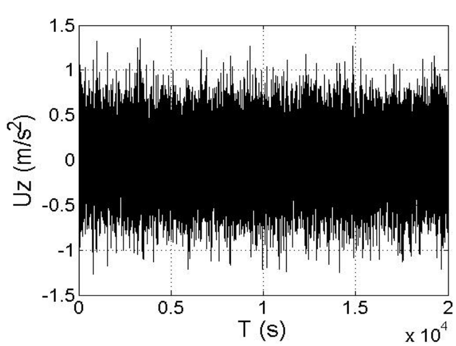

Fig. 16: Z component of the control acceleration without saturation

6. CONCLUSION

The results show that the mentioned algorithm using the actuator and the sensors, are capable of orbit keeping in the orbit plane and in the normal direction. In the orbit plane, this is shown by keeping the satellite altitude (orbit size) and eccentricity (orbit shape) and in the normal direction by keeping inclination and right ascension of the ascending node (orbit plane). Figures 14 to 16 show that in order to keep the orbit with the shown accuracy in figures 6 to 9, we should use thrusters with magnitude of ± 2 m/s^2 or ± 200 N. However, when using saturating ± 5 N thrusters, the range of change in orbital parameters increase as shown in figures 10 to 13. Although, the saturation of thruster results in error increase in keeping orbit, this is necessary to model precisely an actuator.

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